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# Cosmic Inflation and Cosmic String

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We discuss the evolution of inhomogeneous space-time with a spinning fluid in higher dimensions. Using these evolving solutions, we explain cosmic inflation and the formation of a gravitational nontopological cosmic string.

## 1. INTRODUCTION

Ray and Smalley (1982) derived the energy-momentum tensor of a fluid with internal spin. Using this result, Som *et al.* (1988) investigated the evolution of an inhomogeneous and anisotropic cosmological model, and Berman (1990) discussed inhomogeneous inflation with a spinning fluid.

Kibble (1976) first proposed the idea of a cosmic string. A cosmic string is an infinitely long line with high mass, formed as a consequence of macroscopic topological defects. According to the grand unified model, phase transitions in the early universe can give rise to the formation of a cosmic string (Vilenkin, 1985). Zeldovich (1980) and Vilenkin (1981) indicated that cosmic strings can play an important role in explaining the formation of galaxies.

In this paper, we give a new way to form a cosmic string. We call a gravitational string one that is a nontopological cosmic string formed by self-gravity. Our model can explain both the inflation of the universe and the formation of a nontopological cosmic string.

## 2. FIELD EQUATIONS IN HIGHER-DIMENSIONAL CASE

We start with an extended Szekeres (1975) class II metric

$$ds^2 = -dt^2 + Q^2(t, x_1, x_a)\, dx_1^2 + R^2(t)(dx_2^2 + \cdots + dx_{D-1}^2) \qquad (1)$$

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where $Q$ and $R$ are functions to be determined, $D$ is the dimension of space-time, and $a=2,\ldots,D-1$.

Ricci tensors for the above metric (1) are given by

$$R_{00}=(D-2)\ddot{R}/R+\ddot{Q}/Q, \qquad R_{11}=\sum_b QQ_{,bb}/R^2-(D-2)Q\dot{Q}\dot{R}/R-Q\ddot{Q}$$

$$R_{bb}=Q_{,bb}/Q-R\ddot{R}-(D-3)\dot{R}^2-R\dot{R}\dot{Q}/Q, \qquad R_{01}=0$$

$$R_{0b}=\dot{Q}_{,b}/Q-Q_{,b}\dot{R}/QR, \qquad R_{ab}=Q_{,ab}/Q$$

The energy-momentum tensor of the spinning fluid is (Ray and Smalley, 1982)

$$T^{ij}=(\rho+S^{kh}\omega_{kh})u^iu^j+[p+S^{kh}\omega_{kh}/(D-1)](g^{ij}+u^iu^j)+S_k^{(i}\omega^{j)k}$$
$$+S_k^{(i}\sigma^{j)k}-g^{ij}S^{kh}\omega_{kh}/(D-1)+q^{(i}u^{j)} \qquad (2)$$

where

$$q^i=S^{ik}\dot{u}_k+S^{ik}_{;k}-S^{kh}\omega_{kh}u^i \qquad (3)$$

$i, j, k, h=0,1,\ldots,D-1$. The $S^{ij}$ are the spinning tensors, and $\omega_{ij}$ are the spin vector angular velocities.

In moving coordinates, $u^0=-u_0=1$, and the rest are $u_i=u^i=0$ $(i\neq 0)$; we easily get

$$\sigma^{0i}=0, \qquad S^{ii}=0, \qquad \omega^{ij}=0 \qquad (4)$$

The only nonnull components with spin in $T^{ij}$ are $T^{0b}$ and $T^{0b}=S^{bc}_{;c}$. Thus, the Einstein equations are given by

$$(D-2)\dot{Q}\dot{R}/QR+(D-2)(D-3)(\dot{R}/R)^2/2-\sum Q_{,bb}/QR^2=\rho \qquad (5)$$

$$-(D-2)\ddot{R}/R-(D-2)(D-3)(\dot{R}/R)^2/2=p \qquad (6)$$

$$-(D-3)\ddot{R}/R-(D-3)\dot{R}\dot{Q}/RQ-(D-3)(D-4)(\dot{R}/R)^2/2$$
$$-\ddot{Q}/Q+\sum_{c\neq b} Q_{,cc}/QR^2=p \qquad (7)$$

$$\dot{Q}_{,b}-Q_{,b}\dot{R}/R=R(QS^{bc})_{,c} \qquad (8)$$

$$Q_{,ab}=0 \qquad (9)$$

Similar to Berman (1990), we take $S^{bc}$ to be of the following form:

$$S^{bc}=h^{bc}(x_1)/Q \qquad (10)$$

Here $S^{bc}$ are the only nonnull components, and $h^{bc}(x_1)$ are arbitrary functions of $x_1$.

Combining (8) with (10), we have

$$\dot{Q}_{,b}/Q_{,b}=\dot{R}/R \qquad (11)$$

The solution of equation (11) is given by

$$Q_{,b}=f(x_1, x_a)R(t) \tag{12}$$

Integrating (12), we get

$$Q=k(x_1, x_a)R(t)+g(x_1, t) \tag{13}$$

where $k_{,b}=f$, $g$ is an arbitrary function of $x_1$ and $t$.

From (7), we easily prove

$$Q_{,cc}=Q_{,bb} \tag{14}$$

Considering (9), (13), and (14), we can take the $k(x_1, x_a)$ to be

$$k(x_1, x_a)=M(x_1)\sum_b\left(\tfrac{1}{2}\lambda x_b^2+C_b x_b+D_b\right) \tag{15}$$

and we assume

$$g(x_1, t)=M(x_1)\mu(t) \tag{16}$$

where $\lambda$, $C_b$, $D_b$ are constants.

Substituting (13) into (5), we have

$$\left[\frac{1}{2}(D-1)\frac{\dot{R}^2}{R}-\frac{\rho R}{D-2}\right]\frac{k}{M}+\frac{\dot{\mu}\dot{R}}{R}-\frac{\lambda}{R}+\frac{1}{2}(D-3)\frac{\mu\dot{R}^2}{R}-\frac{\mu\rho}{D-2}=0 \tag{17}$$

If $p$ is a function only of $\rho$, from (6) we see that $\rho$ is only related to $t$. It must be pointed out that $k/M$ is only related to $x_a$; in (17) the parts including $k/M$ are separated from those terms including time $t$. For $k/M=$ const, we have

$$\tfrac{1}{2}(D-1)\dot{R}^2/R-\rho R/(D-2)=0 \tag{18}$$

$$\dot{\mu}\dot{R}/R-\lambda/R+\tfrac{1}{2}(D-3)(\dot{R}/R)^2\mu-\mu\rho/(D-2)=0 \tag{19}$$

Equations (18) and (19) can be rewritten as

$$\rho=\tfrac{1}{2}(D-1)(D-2)(\dot{R}/R)^2 \tag{20}$$

$$\dot{\mu}\dot{R}/R-(\dot{R}/R)^2\mu-\lambda/R=0 \tag{21}$$

## 3. INFLATIONARY SOLUTIONS AND STRING SOLUTION

Now we study evolving solutions in higher-dimensional cases.

1. The case of $\lambda=0$ and $p=-\rho$. From (6) and (20), we have

$$\ddot{R}/R-(\dot{R}/R)^2=0 \tag{22}$$

The solution of (22) is given by

$$R=R_0\exp(Bt) \tag{23}$$

Here $B>0$, and $B$ is a constant.

For $\lambda = 0$, considering (21), we easily get

$$\mu = \mu_0 \exp(Bt) \tag{24}$$

The solutions (23) and (24) are for exponential inflation. This is an extension of the conclusion of Berman (1990). But we cannot get any power-law inflationary solution; in fact, for $R \sim t^n$, we can prove $n < 2/3$.

2. The case of $\lambda \neq 0$. From (6) and (7), we have

$$\ddot{\mu}/\mu = \ddot{R}/R \tag{25}$$

Equations (21) and (25) have two kinds of solutions; for $R = t^n$, equations (21) and (25) become

$$\dot{\mu} = nt^{-1}\mu + \lambda t^{1-n}/n \tag{26}$$

$$\ddot{\mu} = \mu n(n-1)t^{-2} \tag{27}$$

Solving (26) and (27), we get

$$\mu = \frac{\lambda(n-1)t^{n+2}}{2n^2} + Ct^n \tag{28}$$

For $R = \exp(-Bt)$, we easily get

$$\mu = -\lambda \exp(Bt)/2B^2 \tag{29}$$

If $n = -1$, or $B > 0$, then $R^2 \to 0$ and $Q^2 \to \infty$ when $t \to \infty$; the results show that the space of $x_a$ dimensions will almost disappear, but the space of dimension $x_1$ remains. A cosmic string has evolved from the fluid by self-gravity. This is a nontopological cosmic string. If $n > 1$, then the above solution is a power-law inflationary solution.

## 4. CONCLUSION

In higher-dimensional space-time, adopting an inhomogeneous model, we find various evolving solutions of a fluid with spin. Several of them may be used to explain cosmic inflation, including exponential and power-law forms. In particular, we get a nontopological cosmic string that has evolved from the fluid with spin by self-gravity. We call the string a gravitational string; evidently the string is an open one.

## REFERENCES

Berman, M. S. (1990). *Nuovo Cimento B*, **105**, 235.
Kibble, T. W. B. (1976). *Journal of Physics A*, **9**, 1387.
Ray, J. R., and Smalley, L. L. (1982). *Physical Review D*, **26**, 2819.
Som, M. M., Beldran, M. L., and Vasconcellos-Vaidya, E. P. (1988). *Nuovo Cimento B*, **102**, 573.
Szekeres, P. (1975). *Communications in Mathematical Physics*, **41**, 55.
Vilenkin, A. (1981). *Physical Review Letters*, **46**, 1169.
Vilenkin, A. (1985). *Physics Reports*, **121**, 263.
Zel'dovich, Ya. B. (1980). *Monthly Notices of the Royal Astronomical Society*, **192**, 663.